Aiming for Understanding:

Lessons Learned about Writing in Mathematics

Finding ways to help students effectively communicate their mathematical thinking is a challenge for teachers across the country. Through a Professional Development Schools (PDS) partnership between the University of Maryland and several elementary schools in a neighboring district, school and university partners worked together to explore, discuss, and reflect on the challenges of helping students write about mathematics. Along with gaining insights into effective instructional strategies, partners from both institutions gained a deeper appreciation for the benefits of school-university dialogue.

Beginning with Student Data

The focus on writing about mathematics began during PDS summer planning meetings prior to the start of the school year. The school and university partners analyzed current student achievement data, leading to the creation of a PDS professional development plan for the upcoming year. This plan addressed ways in which the partnership could assist schools with their identified needs. Mathematics became the focus of that plan, with the specific target areas of developing students' reasoning and problem-solving skills in conjunction with their written communication skills. Using input from the planning meetings, a university course was developed to address these specific mathematics topics, and the course was designed to include both theoretical foundations and practical classroom applications. Along with regular class meetings during the fall semester, electronic communication was initiated to further link the twenty-two participating teachers across the five schools and to allow for ongoing sharing of ideas throughout the school year. As the year closed, a subgroup of participants continued discussions through a series of meetings focused on identifying progress and sharing insights about their classroom experiences.

The Journey from Ideas to Practice

During the mathematics course taught by the university PDS coordinator, teachers explored a variety of strategies for developing students' writing about mathematics. They discussed ideas for expanding students' mathematics vocabularies, explored foundational skills for effective communication, examined the role of teacher modeling, discussed the value of partner and group activities, and explored strategies for providing specific feedback to improve student writing. Teachers were required to try activities with their elementary students and then return to the group with observations and insights.

Teachers were asked to begin regular writing tasks with the students in their classrooms and to have their students record their writing in mathematics journals that could be analyzed throughout the course. As the semester began, writing in mathematics was viewed by most of the participating teachers as simply an assignment for their students

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to complete. Although the teachers asked their students to write about mathematics, they gave little instruction or support to students in the writing endeavor. As participants reviewed their students' efforts and began to gain new ideas and discuss theories, they began to provide students with support, such as graphic organizers, demonstrations and modeling, tips for improving written work, and specific lessons on ways to more effectively explain or describe ideas in writing. Several times during the fall semester, teachers brought their students' mathematics journals to class and shared them with one another, discussing strengths that the students were displaying as well as compiling lists of items that needed improvement. As the semester ended, the participants were encouraged to continue the journal-writing activities with their students for the remainder of the school year.

During the spring semester, a subgroup of teachers from the class decided to continue their review of student work and their reflection on student progress. The university coordinator visited classrooms to demonstrate techniques, coteach lessons, and engage in discussions about students and their learning. As the academic year came to a close, this subgroup of teachers met to review the yearlong effort. Again, with student journals in hand, this partnership team shared student work samples, discussed the progress students had made, vented ongoing frustrations, and expressed ways in which they planned to modify their instruction for the following year. Their participation in university coursework introduced new strategies and ideas; however, their analyses of students' work and the ongoing discussions and reflections with their peers significantly contributed to their development of new attitudes and understandings about the role of writing in their mathematics classrooms. The group discussions allowed participating teachers and the university coordinator to share experiences and observations, refine and clarify ideas about effective teaching strategies, brainstorm solutions, and celebrate successes. The observations and experiences of this partnership team evolved into the following "lessons learned" that have affected their mathematics teaching and the role of writing in their mathematics classrooms.

Lessons Learned: Comments from Our School-University Partnership Team

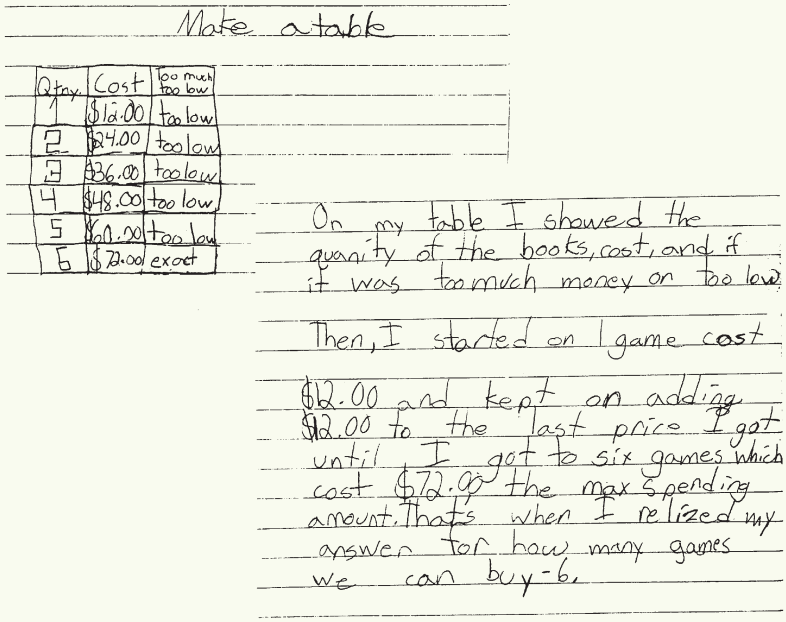
Lesson 1: Sharing ideas benefits written expression

"Group work just takes too much time!" We admitted thinking this way as we struggled to teach innumerable mathematics concepts to our students in a very limited amount of time. We had come to believe that we could cover more material if we

Figure 1

This student chose the make-a-table strategy to help figure out the answer.

"Our class would like to buy board games to use during indoor recess and class parties. For one game, the cost is $12.00. For two games, the cost is $24.00, and for three games, the cost is $36.00. If we have a total of $72.00 to spend, how many games can we buy? Solve and explain your answer."



taught students rather than opened our classes to frequent group and partner activities. As we thought about the connection between oral and written expression, we began to conjecture that *talking about mathematics* might actually help students *write about mathematics*. We began to include more discussions and group work in our daily lessons, and then we observed what happened in our classrooms.

Classroom experiences

Through coteaching opportunities with the university coordinator, fifth-grade teacher Jennifer Beyea observed the benefits of think-pair-share activities (Lyman 2003) and began to incorporate them into her mathematics lessons on a regular basis. This quick cooperative learning technique calls for the teacher to pose a question for individual quiet reflection (think), then asks students to talk to a partner about the question (pair), and then invites students to share their ideas in a class discussion (share). Introducing think-pair-share allowed students of all ability levels in Beyea's class to be involved in her lessons. Even reluctant students began to share their ideas in the nonthreatening pair environment. Beyea recalled observing a student daydreaming in class during the "think" portion of one task and wondering how he would be able to pair with his partner to discuss the problem. After sitting for a minute and not being able to say anything to his partner, he turned to Beyea and asked whether he could have some more "think time." She gave him some time to think and then paired him again with his partner to discuss the problem. When the time came to share ideas with the class, his hand was the first to go up, wanting to share the ideas that he and his partner had discussed. Beyea was thrilled with the involvement of this usually reluctant student. She noted that along with her students' being more involved in discussions, they also now had more ideas to write about when she asked them to record their ideas.

As students in Nikki Friedland's fourth-grade classroom worked with partners to identify the steps to solving problems, she noticed students assisting each other in their understandings. Working together enabled them to analyze their thinking and allowed them to test their thinking for accuracy and reasonableness. As she observed one pair, composed of an average and a struggling student, she "was amazed at the growth in both students' writing. Not only was my struggling student able to express himself more clearly, but my average student's writing became more organized and more thorough as a result of having to carefully explain her thinking to her partner."

Our reflections

We heard comments such as, "Wait—that's not what I meant to say. I meant . . . " as our students caught their own mistakes as they verbalized and then refined their ideas. We heard students say, "Oh, I get it now!" as they listened to others' ideas and incorporated those ideas into their own understanding. Simply, they helped one another refine their thoughts and worked together to find the words to express those thoughts.

Lesson 2: Writing is a tool for learning

We quickly accepted the idea that reading students' writing about how they solved a mathematics problem might help us better assess their problem-solving skills. Without explanations, we were often unable to determine whether students actually knew how to solve the problem, particularly when their solutions were incorrect. To get a better idea

of their development as thinkers, we began to require writing about problem solving.

Classroom experiences

At the beginning of the year, teacher Leila Dowdall began introducing weekly problem-solving activities to her fourth-grade students. The students explored their own thinking through class discussions and group work. Dowdall used think-alouds to model her own problem-solving thinking and engaged students in discussions about their strategies. In addition, writing activities were included to allow her to assess her students' progress. Dowdall introduced vocabulary that allowed students to effectively communicate about their thinking (for example, *strategies, diagram, logical reasoning, pattern*). As the year progressed, she noticed her students' vocabularies expanding to include a variety of words and terms that helped them put their thinking into words as they explained their problem-solving processes and justified their solutions.

By mid-year the students were solving problems using a variety of strategies and were writing effectively about their thinking. During classroom visits, the university coordinator observed the students identifying their thinking strategies and justifying their choices. Dowdall found that, after repeated sharing of student work, her students began to realize that the same problem could be solved using different strategies, and they took great interest in the various strategies used by their classmates. In **figure 1,** the student chose to create a table to display data and explained how the table helped him realize when he had arrived at an answer. The student in **figure 2** chose to represent her work in a different way, listing the multiplication facts that she had tried as she searched for a solution. Students shared their writing after the task and were interested in how others had arrived at their solutions.

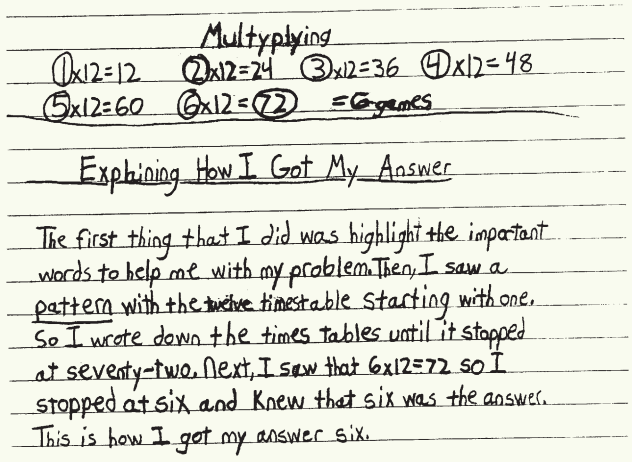
Our reflections

Writing during problem-solving instruction was introduced to help us assess our students' thinking, but as we observed our students and analyzed their work, we concluded that writing about problem solving and reading others' thoughts about problem solving actually helped our students develop a deeper understanding of the problem-solving process. What began as an assessment tool was now being recognized as an instructional tool. We were beginning to view writing as integral to the teaching of mathematics concepts.

Figure 2

Noticing a pattern in the multiplication table helped this student find the answer.

"Our class would like to buy board games to use during indoor recess and class parties. For one game, the cost is $12.00. For two games, the cost is $24.00, and for three games, the cost is $36.00. If we have a total of $72.00 to spend, how many games can we buy? Solve and explain your answer."



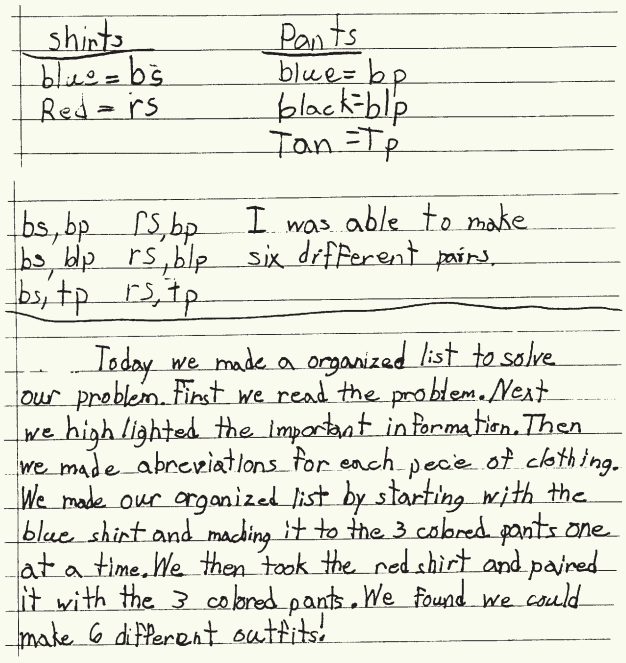
Lesson 3: Practice does not always make perfect

"Explain your answer" is a phrase frequently heard in our mathematics classrooms. We asked students to explain their thinking and were often frustrated when the writing did not meet our expectations. As we met and discussed our students' writing and reflected on the quality of their work, we began to reflect on our instructional practices, realizing that we were *assigning* writing more than we were *instructing* about writing. We realized that our students often had ideas about mathematics concepts but struggled to find the words to express those ideas. As we observed our students more closely and interacted with them about their mathematical understandings, we began to acknowledge that the level of our students' success in writing about mathematics was determined by both their ability to think mathematically and their ability to express that thinking in writing. In analyzing our own instructional practices, we realized we were not providing the same instruction and support to our students when we asked them to write about mathematics as when we asked them to write in other content areas. We noted that we frequently modeled for students during language arts lessons as we taught them how to write explanations, descriptions, or comparisons but that we rarely used the

Figure 3

This student explains the steps he used to solve the problem by using sequence words.

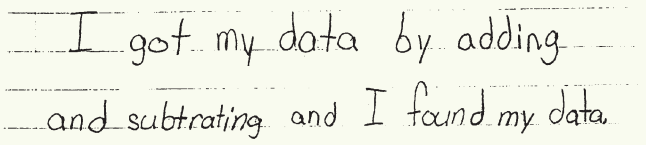
"Yesterday I went to Old Navy to buy some clothes. When I got home I opened my bag and this is what I saw: a blue shirt, a red shirt, one pair of black pants, one pair of blue pants, and one pair of tan pants. How many different outfits could I make with the clothes I bought?"



same technique during mathematics instruction. The group decided to begin helping students write about mathematics, using such techniques as modeling effective mathematics writing, helping students organize their thoughts prior to writing, and providing support for students who struggled in their writing attempts. Along with demonstrating effective thinking skills, we began to demonstrate ideas for how to put that thinking into writing.

Figure 4

This student's writing is vague and does not adequately describe her thought processes.



Classroom experiences

Celia Beamon's fourth-grade students struggled to explain their thinking as they solved problems. During coteaching opportunities with the university coordinator, Beamon was able to generate ideas for helping her students identify and express their thinking during problem solving. The pair decided to try the use of a sequence-chain graphic organizer to help students break down the problem-solving process into clear and ordered steps. Modeling with a sample problem, they demonstrated how to identify and record the steps in solving the problem and then paired the students to identify steps for a similar problem. As students worked together to solve problems and record their thinking on the sequence chains, the coteachers observed the students' interactions and writing. The ideas recorded on their sequence chains became the foundation that the students needed to write organized paragraphs about how they solved the problems. Beamon noted that the paragraphs were more ordered and more detailed than in her students' previous writing.

Early in the year, students in Susan Denvir's second-grade classroom struggled in their attempts to write answers to open-ended questions about mathematics. In an effort to provide more support for the students, she began using sentence starters ("I know that is the answer because . . . ") and closure techniques ("Some even numbers are ___. I know they are even because ___.") to help her students construct their responses. This technique supported the students early in the year, and as the year progressed, the students were better able to write their responses without these supports.

Answering open-ended questions with reasonable and related answers was a stumbling point for students in Nikki Friedland's fourth-grade classroom. She introduced QPA (Question as Part of the Answer) to her students to help them get started on their responses. After modeling this strategy, one student, Nicole, changed her opening sentence from "The answer is 10:30" to "If it's 7:30 now, in 3 hours it will be 10:30" and then proceeded to explain her process for figuring out the answer. This simple technique not only helped jump-start Friedland's students who struggled with how to begin their response but also focused students' attention on the question being asked, helped students construct better opening sentences, and allowed students to self-check the reasonableness of their answers as they wrote both question and answer together. After rereading his sentence "If it is 7:30 now, it will be 4:30 in 3 hours," Trey immediately

recognized the unreasonableness of his answer.

In Leila Dowdall's fourth-grade classroom, students' writing often lacked organization, and she sometimes had difficulty following students' thought processes. Dowdall showed students how to use numbered lists to organize their thoughts and then helped them convert their lists into organized paragraphs using such sequence words as *first, then, next,* and *finally*. In **figure 3,** the student is able to explain the steps he used to solve the problem by using sequence words to support his writing.

Our reflections

"Explain your answer" no longer causes stress or confusion for students in our classrooms. Teacher guidance, modeling as appropriate, and providing instructional supports have empowered our students to be able to more clearly and more thoroughly express their thinking.

Lesson 4: A hidden target is hard to hit

As we analyzed students' writing, we began to wonder whether students really knew what we were asking them to do. When asked to explain how they got an answer or to justify their answer, students often responded in unexpected ways. After solving a problem and being asked to explain how she did it, one student, Jenna (see **fig. 4**), said, "I got my data by adding and subtracting and I found my data." We had hoped that Jenna would provide more details, telling us which numbers she added or subtracted and why she chose those operations, but Jenna's writing was vague and did not adequately describe her thought processes. We decided that we would clarify our expectations through introducing rubrics and providing specific feedback on student work.

Classroom experiences

We began to establish expectations prior to the writing task by introducing the scoring rubric. The students in Celia Beamon's fourth-grade classroom were encouraged to refer to the rubric throughout the writing process. She observed, "My students began to know exactly what I was looking for in their work. They understood that there was more to a quality response than just the right answer. The rubric helped them see that explaining how they solved it was as important as finding the correct answer." After the writing task was completed, Beamon utilized both poor and exemplary samples to further help students understand the expectations of the lesson. She placed samples of exemplary student work on an overhead transparency, as well as examples of poorly written responses that she had generated for the purpose. The class critiqued the samples, with the teacher sharing specific ideas for improving the work or specific comments on what the student had done well. As the teacher and students discussed the piece of writing in **figure 5,** the teacher made notations on the transparency to record their feedback and revision ideas. The teacher underlined and circled key vocabulary words that the student used in his writing, and recorded students' suggestions for improving the work. By the end of the year, when students

Figure 5

The teacher makes notations on an overhead transparency to record students' feedback.

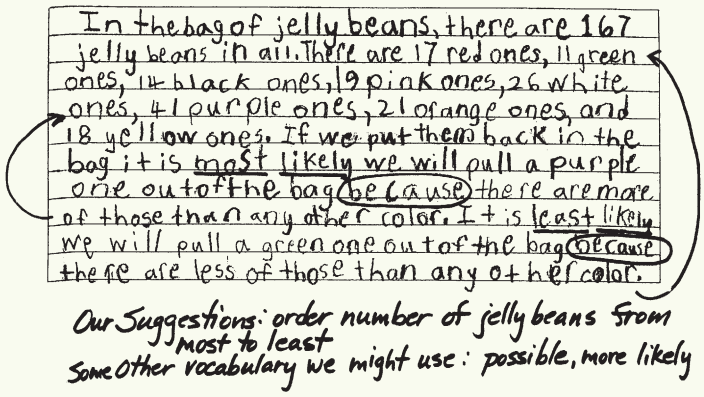
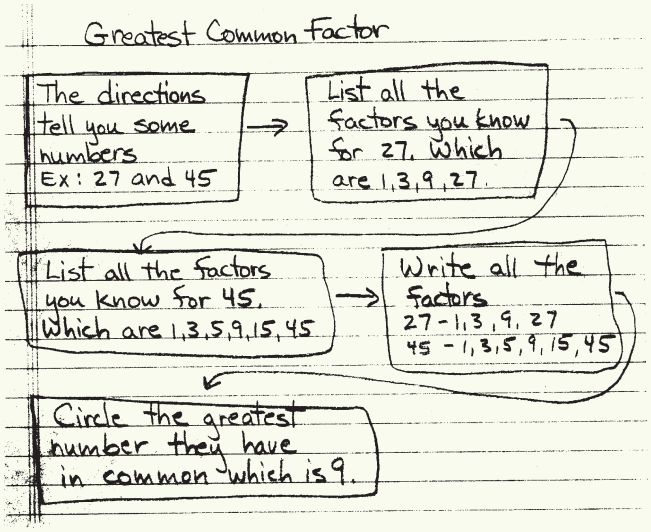


Figure 6

Using sequence chains helped students identify the steps in finding the greatest common factor.



Reflect and Discuss: *Aiming for Understanding: Lessons Learned about Writing and Mathematics*

Reflective teaching is a process of self-observation and self-evaluation. It means looking at your classroom practice, thinking about what you do and why you do it, and then evaluating whether it works. By collecting information about what goes on in our classrooms, then analyzing and evaluating this information, we identify and explore our own practices and underlying beliefs.

The following questions related to "Aiming for Understanding: Lessons Learned about Writing in Mathematics" by Susan R. O'Connell, Celia Beamon, Jennifer M. Beyea, Susan S. Denvir, Leila A. Dowdall, Nikki G. Friedland, and Joelle D. Ward are suggested prompts to aid you in reflecting on the article and on how the authors' idea might benefit your own classroom practice. You are encouraged to reflect on the article independently as well as discuss it with your colleagues.

- **In what ways do your students struggle as they attempt to write about mathematics? Prioritize a list of potential strategies that you gained from this article and other resources that you could use to assist them.**
- **What can you do to persuade reluctant writers that writing about their mathematical thinking and problem-solving processes is worth their time and effort?**
- **In what ways do inquiry, discussion, and reflection promote professional growth? How might you incorporate more reflections and collaboration into your daily instructional routine?**

The Editorial Panel appreciates the interest and values the views of those who take the time to send us their comments. Letters may be submitted to Teaching Children Mathematics, *"Reader's Exchange," 1906 Association Dr., Reston, VA 20191-1502; or send electronic submissions to tcm@nctm.org. Because of space limitations, letters and rejoinders from authors beyond the 250-word limit may be subject to abridgement. Letters also are edited for style and content.*

were asked to praise good qualities in a piece of work and offer ideas to improve the piece, they could be heard commenting on word choice, clarity of ideas, thoroughness, and sequence. They had internalized the expectations for their responses.

Our reflections

As students began to understand what was expected in their writing, their writing improved. Introducing the rubric to students prior to the task, providing specific feedback on student work, sharing samples of exemplary work, and having students use rubrics to score their own or others' work were all ways in which we communicated our expectations to students.

Lesson 5: Writing about mathematics helps students hit the bull's-eye

Through our group discussions, we began to realize the large amount of time we were devoting to teaching how to do a skill rather than understanding that skill and when and how it would be useful to students. As we began to add writing to our mathematics lessons, we realized it continually brought us back to a focus on understanding. Whether we were asking students to explain a process, justify an answer, describe a pattern, or list real-world connections, we found that writing focused us on students' understanding of mathematics concepts.

Classroom experiences

Sixth-grade teacher Joelle Ward found that some of her students were having difficulty distinguishing between the least common multiple and the greatest common factor. In an effort to strengthen their understanding of the concepts, she turned to some writing tasks that focused on the two mathematics concepts. First, students were asked to work with a partner to compare and contrast the concepts using a Venn diagram. This tactic allowed students to analyze the two concepts and reflect on their similarities and differences. Students created sequence chains to identify the steps in finding the greatest common factor of two numbers (see **fig. 6**). After creating sequence chains for finding the least common multiple, students were able to compare the processes that had confused them. These writing experiences helped Ward's students develop a clearer understanding of these concepts.

Our reflections

Incorporating writing into our mathematics lessons helped us stay focused on teaching for understanding. As we reviewed our students' work, we were able to see the level to which they understood each concept, something a correct answer had not always conveyed to us. We discovered that our students' writing focused on the how and why of mathematics, not just the answer, and helped our students connect mathematics with their own lives.

The Journey Continues: Our Final Reflections

Over the course of one academic year, our school-university collaboration provided elementary teachers with opportunities to gain new ideas, try new strategies, and reflect on students' abilities. In

addition, it offered the university coordinator ongoing opportunities to work collaboratively with practicing teachers and to interact with and observe elementary school students as they developed mathematical understandings. Through a process of trying research-based techniques, observing student interactions, analyzing student work samples, and engaging in reflective discussions, both school and university partners expanded their knowledge of writing about mathematics. Together, we gained new insights into the challenges and successes of communicating about mathematics.

Through this process of inquiry, practice, and reflection, those of us who are classroom teachers modified our attitudes and practices related to writing about mathematics. Writing about mathematics is now an integral component of our elementary mathematics programs, not because we have been told that it should be but because we have seen the benefits for our students. In addition, opportunities to coteach in elementary classrooms and discuss crucial issues with practicing teachers have enhanced the university program and provided concrete experiences to support educational theory.

We are enthusiastic about our discoveries related to student writing about mathematics. This approach offers challenges on a daily basis, but we now have a greater understanding of how to meet many of those challenges and are willing to continue to reflect on those solutions we have not yet discovered. As important as our discoveries related to writing about mathematics is our discovery of the value of school-university partnerships for fostering a medium for ongoing and substantive reflection on our practices. Only as we stepped back and reflected on our practice did we gain crucial insights that have enhanced our teaching of mathematics and our students' understanding of mathematics.

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