Progression of Number Skills, Concepts and Mental Math Strategies
PreK through Fifth Grade

Beginning Math Skills

Counting: (PreK)

- saying the numbers in order
- counting objects using the counting sequence (using a number for each item counted)
- knowing that the last number you count when counting a set of objects tells the amount of objects in the set

Beginning Addition: (PreK-1)

- double counts
  - 5 + 3: child counts out 5 and then counts out 3, counts all 8 to find the total
  - sounds like 1, 2, 3, 4, 5; 1, 2, 3; 1, 2, 3, 4, 5, 6, 7, **8**
- counts on
  - 5 + 3: Counts out (or puts out) 5 and then counts on while adding the next 3
  - sounds like 1, 2, 3, 4, **5**, 6, 7, **8**
- recognizes a group 5 and 3 as **8** without counting

NOTE: As numbers get larger or out of a student's comfort level, most children will tend to revert back to counting, often by ones.

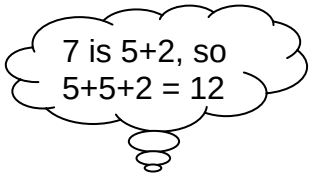
Number (facts) Relationships: (K-2) Many students begin to memorize the addition facts in the following sequence:

- doubles (1+1, 2+ 2, 3 + 3, 4+ 4, 5 + 5, 6 + 6, 7 + 7, 8 + 8, 9 + 9)
- doubles plus 1: Child uses what they know about doubles and adds 1
  - sounds like 8 + 9 is the same as 8 + 8 +1 which is 9
- doubles minus 1: Child uses what they know about doubles and subtracts 1
  - sounds like 8 + 7 is the same as 8 + 8 -1
- number plus 1 or plus two (5 + 1, 4 + 2)
- number plus or minus zero (5 + 0, 6 + 0)
- commutative property (3 + 2 = 2 + 3) Children will often refer to this as the “flip flop” rule
- ten combinations (5 + 5, 6 + 4, 7 + 3, 8 + 2, 9 + 1)
- ten plus a number (10 + 4, 10 + 7)

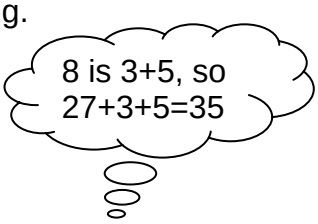
Mental Math Strategies: (1st – 2nd Grade)

Composing or decomposing numbers (splitting numbers) to make a ten or a double: (1st-2nd grade)

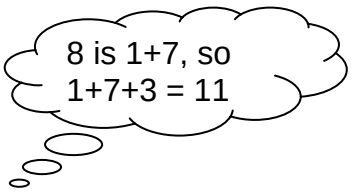
The child finds the sum of the numbers mentally, by decomposing one of the numbers to allow them to make the problem easier to add mentally. The numbers inside the “bubbles” indicate what the child might be thinking.



5 + **7** = 12

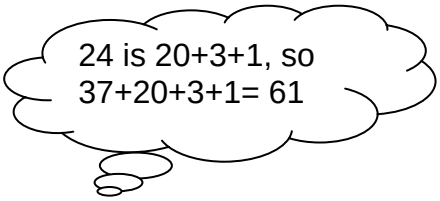


27 + **8** = 35

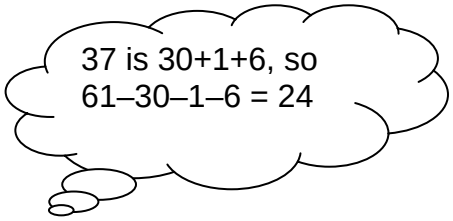


8 + 3 = 11

Keeping one number whole and decomposing the other number with 2-digit numbers:

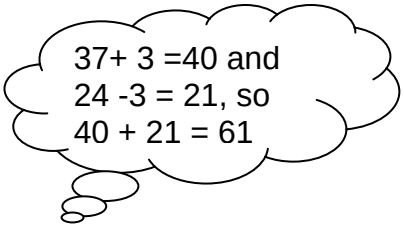


37 + **24** = 61

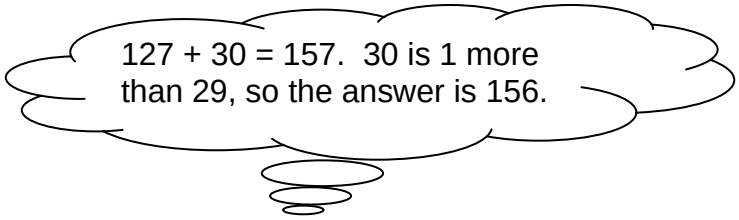


61 – **37** = 24

Compensation:

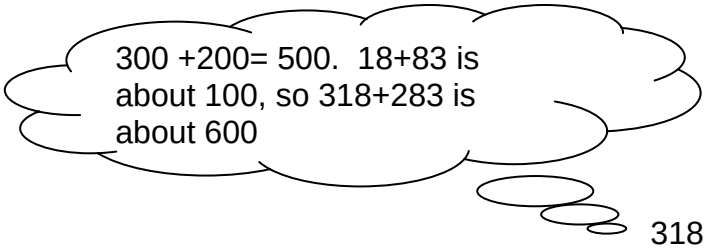


37 + 24 = 61



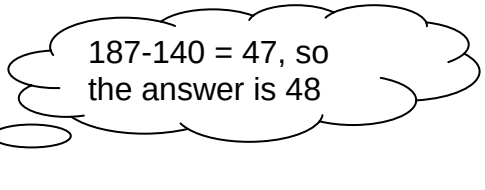
127 + **29** = 156

Front-end Estimation and Compensation (Grades 3-5):

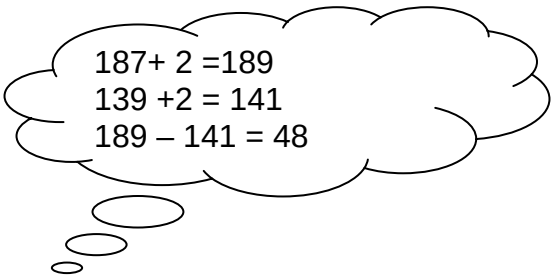


318
<u>283</u>
~600

Changing the problem to an easier one:



187 - 139 =



187 - 139 = 48

Beginning with kindergarten students will see and be expected to begin to use both the horizontal (equation format) and vertical (working form format) standard notations for addition and subtraction.

Horizontal: **3 + 7 = 10** **398 + 285 = 683** **5 – 3 = 2** **547 – 197 = 350**

Vertical:

3	1 1 398	5	4 14 5 ~~4~~7
+ 7	+285	- 3	- 197
10	683	2	350

Multiplication

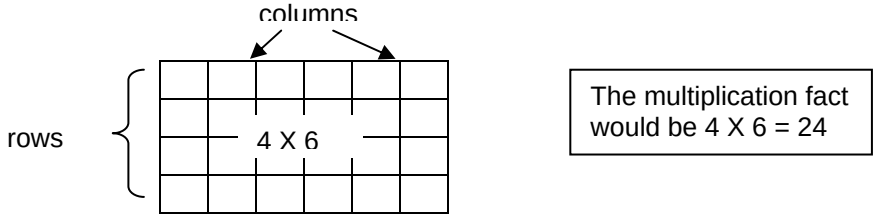
Multiplication is formally studied beginning in 3rd grade. However, much of the ground work for multiplication is introduced in the primary grades.

Things that come in groups (K-3)

- number of legs on a cat, dog
- number of legs on a chair
- number of wheels on a car
- number of cans of soda in a carton
- number of eggs in a dozen
- number of fingers on one hand
- number of eyes

Generally students will begin using repeated addition or skip counting when thinking about multiplication as a grouping model.

Arrays (rectangular) (3-5) are a formation of objects in a rectangle. This method is also sometimes called an area model of multiplication.



The multiplication fact would be 4 X 6 = 24

Using known facts - one of the ways in which students begin to memorize their multiplication facts is to use what they already know to remember a new fact. They generally apply one of the following properties of multiplication - commutative, associative or distributive.

Commutative property: 4 X 5 = 5 X 4

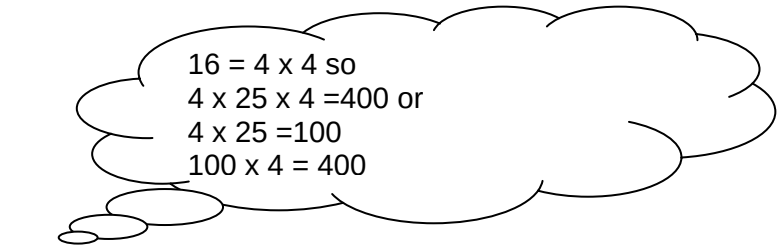
Associative property: (3 X 2) X 5 = 3 X (2 X 5)

Distributive property: 4 X 7 = 4 (5 + 2) = (4 X 5) + (4 X 2)

I know that 4 X 7 = 28 because I know that 4 X 5 = 20 and 4 X 2 = 8

Mental Math Strategies for Multiplication

In solving **16 X 25** a student might split the numbers into numbers that are easier to multiply and then use the associative property of multiplication to regroup the factors.



16 X 25

16 x 25: Using the distributive property the student might think (10X 25) + (6 X 25)

250 + 150 = 400

Beginning in 3rd grade students will be introduced and expected to use the standard notation in both horizontal and vertical notion for multiplication.

Horizontal: **3 x 5 = 15** **25 x 10 = 250**

Vertical:

3	**25**	**156**
x 5	**x 10**	**x 27**
15	**00**	**1092**
	250	**3120**
	250	**4212**

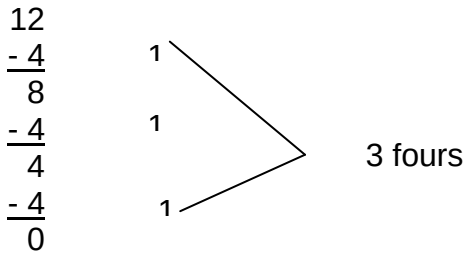
Division is formally studied beginning in 3rd grade.

Students will often begin solving division problems by using repeated subtraction or by dealing out to each group until there are none left or aren't enough left to distribute. They may also use a known multiplication fact to solve an unknown fact.

Dealing out: 12 divided by 4 The child distributes 1 to each group until there are none left.

Repeated subtraction:

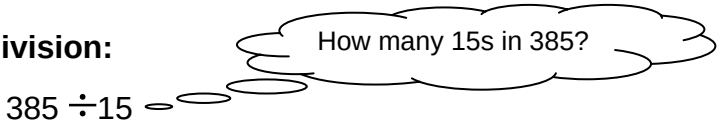
Using a known fact:
I know that 12 divided by 4 = 3
because I know that **4 X 3 = 12**



Long Division

Before dealing with the standard notation of long division, **some** children will use some of the strategies shown below as they are being introduced to the standard long division algorithm.

Larger division:



385
- 150 (10 X 15) 10 + 10 + 5 = **25 r 10**
235
- 150 (10 X 15)
85
- 75 (5 X 15)
10

15	385	25 R10 or 25 $^{10}/_{15}$ or 25 $^{2}/_{3}$
-	150	10 (10 x 15)
	235	
-	150	10 (10 x 15)
	85	
-	75	5 (5 x 15)
	10	25

Beginning in 4th grade students will be introduced to the standard notation for division. They will be expected to master the standard notation for long division by the end of 5th grade.

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